

Pensieve Header: The “ $\$c\to T\$$ ” correction.

```

In[1]:= << KnotTheory`
GD[K_] := GD @@ (
  PD[K] /. X[i_, j_, k_, l_] => If[PositiveQ[X[i, j, k, l]],
    Ar[l, i, +1], Ar[j, i, -1]
  ]
)

Loading KnotTheory` version of August 22, 2010, 13:36:57.55.
Read more at http://katlas.org/wiki/KnotTheory.

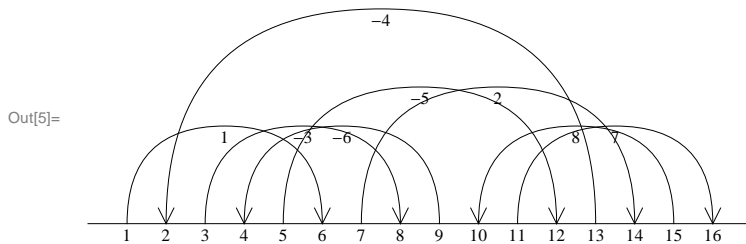
```

```

In[3]:= Draw[expr_] := expr /. gd_GD => Draw[gd];
Draw[gd_GD] := Module[
  {n = Length[gd], h, k = 0},
  Graphics[{
    Line[{{0, 0}, {2 n + 1, 0}}],
    Table[Text[i, {i, -0.3}], {i, 2 n}],
    (List@@gd) /. {
      Ar[i_, j_, s_] => {
        h = Abs[i - j] / 2;
        BezierCurve[
          {
            {i, 0}, {i, h}, {(i + j) / 2, h}, {j, h}, {j, 0}
          }, SplineDegree -> 2],
        Text[s * (++k), {(i + j) / 2, h - 0.3}],
        Line[{{j - 0.2, 0.4}, {j, 0}, {j + 0.2, 0.4}}]
      }
    ]
  ]
];
Draw[GD[Knot[8, 17]]]

```

KnotTheory::loading : Loading precomputed data in PD4Knots`.



```

In[6]:=  $\beta$ Simplify = Factor;
SetAttributes[ $\beta$ Collect, Listable];
 $\beta$ Collect[B[ $\omega$ _,  $\mu$ _]] := B[
   $\beta$ Simplify[ $\omega$ ],
  Collect[ $\mu$ , _h, Collect[#, _t,  $\beta$ Simplify] &]
];
(* "L" for "Labels" *)
hL[ $\beta$ _] := Union[Cases[ $\beta$ , h[s_]  $\Rightarrow$  s, Infinity]];
tL[ $\beta$ _] := Union[Cases[ $\beta$ , t[s_] | T $_{s_}$   $\Rightarrow$  s, Infinity]];
dL[ $\beta$ ] := Union[hL[ $\beta$ ], tL[ $\beta$ ]];
SetAttributes[ $\beta$ Form, Listable];
 $\beta$ Form[B[ $\omega$ _,  $\mu$ _]] := Module[
  {tails, heads, mat},
  tails = tL[B[ $\omega$ ,  $\mu$ ]]; heads = hL[B[ $\omega$ ,  $\mu$ ]];
  mat = Outer[ $\beta$ Simplify[Coefficient[ $\mu$ , h[#1] t[#2]]] &, heads, tails];
  PrependTo[mat, t /@ tails];
  mat = Prepend[Transpose[mat], Prepend[h /@ heads,  $\omega$ ]];
  MatrixForm[mat]
];

In[14]:= R[x_, y_] := B[1, (T $_x$  - 1) t[x] h[y]];
Rinv[x_, y_] := B[1, (1 / T $_x$  - 1) t[x] h[y]];
tm[x_, y_, z_][ $\beta$ _] :=  $\beta$  /. {t[x]  $\rightarrow$  t[z], t[y]  $\rightarrow$  t[z], T $_x$   $\rightarrow$  T $_z$ , T $_y$   $\rightarrow$  T $_z$ };
hm[x_, y_, z_][B[ $\omega$ _,  $\mu$ _]] := Module[
  { $\gamma$ x = D[ $\mu$ , h[x]],  $\gamma$ y = D[ $\mu$ , h[y]], M =  $\mu$  /. h[x] | h[y]  $\rightarrow$  0},
  B[ $\omega$ , M + h[z] ( $\gamma$ x +  $\gamma$ y + ( $\gamma$ x /. t[i_]  $\Rightarrow$  1)  $\gamma$ y)] //  $\beta$ Collect
];
swap[x_, y_][B[ $\omega$ _,  $\mu$ _]] := Module[
  { $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ ,  $\epsilon$ },
   $\alpha$  = Coefficient[ $\mu$ , h[x] t[y]];
   $\beta$  = D[ $\mu$ , t[y]] /. h[x]  $\rightarrow$  0;
   $\gamma$  = D[ $\mu$ , h[x]] /. t[y]  $\rightarrow$  0;
   $\delta$  =  $\mu$  /. h[x] | t[y]  $\rightarrow$  0;
   $\epsilon$  = 1 +  $\alpha$ ;
  B[ $\omega$  *  $\epsilon$ , Plus[
     $\alpha$  (1 + ( $\gamma$  /. t[i_]  $\Rightarrow$  1) /  $\epsilon$ ) h[x] t[y],
     $\beta$  (1 + ( $\gamma$  /. t[i_]  $\Rightarrow$  1) /  $\epsilon$ ) t[y],
     $\gamma$  /  $\epsilon$  h[x],
     $\delta$  - (1 /  $\epsilon$ )  $\gamma$  *  $\beta$ 
  ]] //  $\beta$ Collect
];
gm[x_, y_, z_][ $\beta$ _] :=  $\beta$  // swap[y, x] // hm[x, y, z] // tm[x, y, z];
B /: B[ $\omega$ 1_,  $\mu$ 1_] B[ $\omega$ 2_,  $\mu$ 2_] := B[ $\omega$ 1 *  $\omega$ 2,  $\mu$ 1 +  $\mu$ 2];

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{
   $\beta = \mathbf{B}[\omega[T_1, T_2, T_3, T_4], \text{Sum}[\alpha_i[T_1, T_2, T_3, T_4] t[i] h[1], \{i, 4\}]],$ 
   $\beta // \text{tm}[1, 2, 1],$ 
   $t1 = \beta // \text{tm}[1, 2, 1] // \text{tm}[1, 3, 1],$ 
   $t2 = \beta // \text{tm}[2, 3, 28] // \text{tm}[1, 28, 1],$ 
   $t1 == t2$ 
} //  $\beta\text{Form}$ 


$$\left( \begin{array}{cc} \omega[T_1, T_2, T_3, T_4] & h[1] \\ t[1] & \alpha_1[T_1, T_2, T_3, T_4] \\ t[2] & \alpha_2[T_1, T_2, T_3, T_4] \\ t[3] & \alpha_3[T_1, T_2, T_3, T_4] \\ t[4] & \alpha_4[T_1, T_2, T_3, T_4] \end{array} \right),$$



$$\left( \begin{array}{cc} \omega[T_1, T_1, T_3, T_4] & h[1] \\ t[1] & \alpha_1[T_1, T_1, T_3, T_4] + \alpha_2[T_1, T_1, T_3, T_4] \\ t[3] & \alpha_3[T_1, T_1, T_3, T_4] \\ t[4] & \alpha_4[T_1, T_1, T_3, T_4] \end{array} \right),$$



$$\left( \begin{array}{cc} \omega[T_1, T_1, T_1, T_4] & h[1] \\ t[1] & \alpha_1[T_1, T_1, T_1, T_4] + \alpha_2[T_1, T_1, T_1, T_4] + \alpha_3[T_1, T_1, T_1, T_4] \\ t[4] & \alpha_4[T_1, T_1, T_1, T_4] \end{array} \right),$$



$$\left( \begin{array}{cc} \omega[T_1, T_1, T_1, T_4] & h[1] \\ t[1] & \alpha_1[T_1, T_1, T_1, T_4] + \alpha_2[T_1, T_1, T_1, T_4] + \alpha_3[T_1, T_1, T_1, T_4] \\ t[4] & \alpha_4[T_1, T_1, T_1, T_4] \end{array} \right), \beta\text{Form}[\text{True}] \}$$


{
   $\beta = \mathbf{B}[\omega, \text{Sum}[\alpha_{10\ i+j} t[i] h[j], \{i, 2\}, \{j, 4\}]],$ 
   $\beta // \text{hm}[1, 2, 1],$ 
   $t1 = \beta // \text{hm}[1, 2, 1] // \text{hm}[1, 3, 1],$ 
   $t2 = \beta // \text{hm}[2, 3, 28] // \text{hm}[1, 28, 1],$ 
   $t1 == t2$ 
} //  $\beta\text{Form} // \text{ColumnForm}$ 


$$\left( \begin{array}{ccccc} \omega & h[1] & h[2] & h[3] & h[4] \\ t[1] & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ t[2] & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \end{array} \right)$$



$$\left( \begin{array}{ccccc} \omega & & h[1] & & h[3] & h[4] \\ t[1] & \alpha_{11} + \alpha_{12} + \alpha_{11} & \alpha_{12} + \alpha_{12} & \alpha_{12} & \alpha_{21} & \alpha_{13} & \alpha_{14} \\ t[2] & \alpha_{21} + \alpha_{22} + \alpha_{11} & \alpha_{22} + \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \end{array} \right)$$



$$\left( \begin{array}{ccccc} \omega & & & & h[1] \\ t[1] & \alpha_{11} + \alpha_{12} + \alpha_{11} & \alpha_{12} + \alpha_{13} + \alpha_{11} & \alpha_{13} + \alpha_{12} & \alpha_{13} + \alpha_{11} & \alpha_{12} & \alpha_{13} + \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{21} + \alpha_{13} & \alpha_{21} + \alpha_{13} & \alpha_{22} + \alpha_{11} \\ t[2] & \alpha_{21} + \alpha_{22} + \alpha_{11} & \alpha_{22} + \alpha_{21} & \alpha_{22} + \alpha_{23} + \alpha_{11} & \alpha_{23} + \alpha_{12} & \alpha_{23} + \alpha_{11} & \alpha_{23} + \alpha_{12} & \alpha_{21} & \alpha_{23} + \alpha_{12} & \alpha_{21} & \alpha_{23} + \alpha_{22} & \alpha_{23} + \alpha_{11} \end{array} \right)$$



$$\left( \begin{array}{ccccc} \omega & & & & h[1] \\ t[1] & \alpha_{11} + \alpha_{12} + \alpha_{11} & \alpha_{12} + \alpha_{13} + \alpha_{11} & \alpha_{13} + \alpha_{12} & \alpha_{13} + \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{21} + \alpha_{13} & \alpha_{21} + \alpha_{13} & \alpha_{22} + \alpha_{11} \\ t[2] & \alpha_{21} + \alpha_{22} + \alpha_{11} & \alpha_{22} + \alpha_{21} & \alpha_{22} + \alpha_{23} + \alpha_{11} & \alpha_{23} + \alpha_{12} & \alpha_{23} + \alpha_{11} & \alpha_{23} + \alpha_{12} & \alpha_{21} & \alpha_{23} + \alpha_{12} & \alpha_{21} & \alpha_{23} + \alpha_{22} & \alpha_{23} + \alpha_{11} \end{array} \right)$$

 $\beta\text{Form}[\text{True}]$ 

Clear[ $\beta$ ];
 $\beta1 = \mathbf{B}[\omega, h[1] t[1] \alpha + h[2] t[1] \beta + h[1] t[2] \gamma + h[2] t[2] \delta],$ 
 $\beta1 // \text{swap}[1, 1]$ 
} //  $\beta\text{Form}$ 


$$\left\{ \left( \begin{array}{ccc} \omega & h[1] & h[2] \\ t[1] & \alpha & \beta \\ t[2] & \gamma & \delta \end{array} \right), \left( \begin{array}{ccc} (1+\alpha) \omega & h[1] & h[2] \\ t[1] & \frac{\alpha(1+\alpha+\gamma)}{1+\alpha} & \frac{\beta(1+\alpha+\gamma)}{1+\alpha} \\ t[2] & \frac{\gamma}{1+\alpha} & \frac{-\beta\gamma+\delta+\alpha\delta}{1+\alpha} \end{array} \right) \right\}$$


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{
   $\beta = B[\omega, \text{Sum}[\alpha_{10\ i+j} t[i] h[j], \{i, 2\}, \{j, 3\}]],$ 
   $\beta // \text{hm}[1, 2, 1],$ 
   $t1 = \beta // \text{hm}[1, 2, 1] // \text{swap}[1, 1],$ 
   $t2 = \beta // \text{swap}[1, 1] // \text{swap}[2, 1] // \text{hm}[1, 2, 1],$ 
   $\text{First}[t1] = \text{First}[t2],$ 
   $\text{Last}[t1] == \text{Last}[t2] // \text{Simplify}$ 
} //  $\beta\text{Form} // \text{ColumnForm}$ 


$$\begin{pmatrix} \omega & h[1] & h[2] & h[3] \\ t[1] & \alpha_{11} & \alpha_{12} & \alpha_{13} \\ t[2] & \alpha_{21} & \alpha_{22} & \alpha_{23} \end{pmatrix}$$


$$\begin{pmatrix} \omega & & h[1] & & h[3] \\ t[1] & \alpha_{11} + \alpha_{12} + \alpha_{11} & \alpha_{12} + \alpha_{12} & \alpha_{21} & \alpha_{13} \\ t[2] & \alpha_{21} + \alpha_{22} + \alpha_{11} & \alpha_{22} + \alpha_{21} & \alpha_{22} & \alpha_{23} \end{pmatrix}$$


$$\begin{pmatrix} \omega (1 + \alpha_{11} + \alpha_{12} + \alpha_{11} \alpha_{12} + \alpha_{12} \alpha_{21}) & & h[1] & & h[3] \\ & t[1] & \frac{(1 + \alpha_{11} + \alpha_{21}) (\alpha_{11} + \alpha_{12} + \alpha_{11} \alpha_{12} + \alpha_{12} \alpha_{21}) (1 + \alpha_{12} + \alpha_{22})}{1 + \alpha_{11} + \alpha_{12} + \alpha_{11} \alpha_{12} + \alpha_{12} \alpha_{21}} & & \frac{\alpha_{13} (1 + \alpha_{11} + \alpha_{21})}{1 + \alpha_{11} + \alpha_{12} + \alpha_1} \\ & t[2] & \frac{\alpha_{21} + \alpha_{22} + \alpha_{11} \alpha_{22} + \alpha_{21} \alpha_{22}}{1 + \alpha_{11} + \alpha_{12} + \alpha_{11} \alpha_{12} + \alpha_{12} \alpha_{21}} & & \frac{-\alpha_{13} \alpha_{21} - \alpha_{13} \alpha_{22} - \alpha_{11} \alpha_{13} \alpha_{22} - \alpha_{13} \alpha_{21} \alpha_{22} + \alpha_1}{1 + \alpha_{11} + \alpha_{12} + \alpha_1} \end{pmatrix}$$


$$\begin{pmatrix} \omega (1 + \alpha_{11} + \alpha_{12} + \alpha_{11} \alpha_{12} + \alpha_{12} \alpha_{21}) & & h[1] & & h[3] \\ & t[1] & \frac{(1 + \alpha_{11} + \alpha_{21}) (\alpha_{11} + \alpha_{12} + \alpha_{11} \alpha_{12} + \alpha_{12} \alpha_{21}) (1 + \alpha_{12} + \alpha_{22})}{1 + \alpha_{11} + \alpha_{12} + \alpha_{11} \alpha_{12} + \alpha_{12} \alpha_{21}} & & \frac{\alpha_{13} (1 + \alpha_{11} + \alpha_{21})}{1 + \alpha_{11} + \alpha_{12} + \alpha_1} \\ & t[2] & \frac{\alpha_{21} + \alpha_{22} + \alpha_{11} \alpha_{22} + \alpha_{21} \alpha_{22}}{1 + \alpha_{11} + \alpha_{12} + \alpha_{11} \alpha_{12} + \alpha_{12} \alpha_{21}} & & \frac{-\alpha_{13} \alpha_{21} - \alpha_{13} \alpha_{22} - \alpha_{11} \alpha_{13} \alpha_{22} - \alpha_{13} \alpha_{21} \alpha_{22} + \alpha_1}{1 + \alpha_{11} + \alpha_{12} + \alpha_1} \end{pmatrix}$$

 $\beta\text{Form}[\text{True}]$ 
 $\beta\text{Form}[\text{True}]$ 

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```

{
   $\beta = B[\omega, \text{Sum}[\alpha_{10\ i+j} t[i] h[j], \{i, 3\}, \{j, 2\}]],$ 
   $t1 = \beta // \text{tm}[1, 2, 1] // \text{swap}[1, 1],$ 
   $t2 = \beta // \text{swap}[1, 2] // \text{swap}[1, 1] // \text{tm}[1, 2, 1],$ 
   $\text{First}[t1] = \text{First}[t2],$ 
   $\text{Last}[t1] == \text{Last}[t2] // \text{Simplify}$ 
} //  $\beta\text{Form} // \text{ColumnForm}$ 


$$\begin{pmatrix} \omega & h[1] & h[2] \\ t[1] & \alpha_{11} & \alpha_{12} \\ t[2] & \alpha_{21} & \alpha_{22} \\ t[3] & \alpha_{31} & \alpha_{32} \end{pmatrix}$$


$$\begin{pmatrix} \omega (1 + \alpha_{11} + \alpha_{21}) & & h[1] & & h[2] \\ & t[1] & \frac{(\alpha_{11} + \alpha_{21}) (1 + \alpha_{11} + \alpha_{21} + \alpha_{31})}{1 + \alpha_{11} + \alpha_{21}} & & \frac{(\alpha_{12} + \alpha_{22}) (1 + \alpha_{11} + \alpha_{21} + \alpha_{31})}{1 + \alpha_{11} + \alpha_{21}} \\ & t[3] & \frac{\alpha_{31}}{1 + \alpha_{11} + \alpha_{21}} & & \frac{-\alpha_{12} \alpha_{31} - \alpha_{22} \alpha_{31} + \alpha_{32} + \alpha_{11} \alpha_{32} + \alpha_{21} \alpha_{32}}{1 + \alpha_{11} + \alpha_{21}} \end{pmatrix}$$


$$\begin{pmatrix} \omega (1 + \alpha_{11} + \alpha_{21}) & & h[1] & & h[2] \\ & t[1] & \frac{(\alpha_{11} + \alpha_{21}) (1 + \alpha_{11} + \alpha_{21} + \alpha_{31})}{1 + \alpha_{11} + \alpha_{21}} & & \frac{(\alpha_{12} + \alpha_{22}) (1 + \alpha_{11} + \alpha_{21} + \alpha_{31})}{1 + \alpha_{11} + \alpha_{21}} \\ & t[3] & \frac{\alpha_{31}}{1 + \alpha_{11} + \alpha_{21}} & & \frac{-\alpha_{12} \alpha_{31} - \alpha_{22} \alpha_{31} + \alpha_{32} + \alpha_{11} \alpha_{32} + \alpha_{21} \alpha_{32}}{1 + \alpha_{11} + \alpha_{21}} \end{pmatrix}$$

 $\beta\text{Form}[\text{True}]$ 
 $\beta\text{Form}[\text{True}]$ 

```



```
{
  β = B[ω, Sum[α10 i+j t[i] h[j], {i, 4}, {j, 4}]],
  t1 = β // gm[1, 2, 1] // gm[1, 3, 1],
  t2 = β // gm[2, 3, 2] // gm[1, 2, 1],
  First[t1] == First[t2],
  Last[t1] == Last[t2] // Simplify
} // βForm // ColumnForm
```

A very large output was generated. Here is a sample of it:

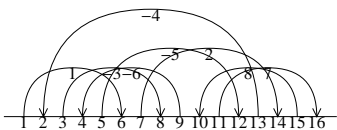
$$\left(\begin{array}{ccccc} \omega & h[1] & h[2] & h[3] & h[4] \\ t[1] & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ t[2] & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ t[3] & \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ t[4] & \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{array} \right)$$

$$\left(\begin{array}{cccc} \omega (1 + \alpha_{12} + \alpha_{13} + \alpha_{12} \alpha_{13} + \alpha_{23} + \alpha_{12} \alpha_{23} + \alpha_{13} \alpha_{32} + \alpha_{13} \alpha_{42}) & & & \\ & t[1] & & \\ & & t[4] & \\ \omega (1 + \alpha_{12} + \alpha_{13} + \alpha_{12} \alpha_{13} + \alpha_{23} + \alpha_{12} \alpha_{23} + \alpha_{13} \alpha_{32} + \alpha_{13} \alpha_{42}) & & & \\ & t[1] & & \\ & & t[4] & \end{array} \right)$$

βForm[True]
βForm[True]

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```
{Alexander[K = Knot[8, 17]][X], Draw[GD[K]]}
```

$$\left\{ 11 - \frac{1}{X^3} + \frac{4}{X^2} - \frac{8}{X} - 8X + 4X^2 - X^3, \right.$$


```
(β = Times @@ GD[K] /. {Ar[x_, y_, +1] => R[x, y], Ar[x_, y_, -1] => Rinv[x, y]}) // βForm
```

$$\left(\begin{array}{cccccccc} 1 & h[2] & h[4] & h[6] & h[8] & h[10] & h[12] & h[14] & h[16] \\ t[1] & 0 & 0 & -1 + T_1 & 0 & 0 & 0 & 0 & 0 \\ t[3] & 0 & 0 & 0 & -\frac{-1+T_3}{T_3} & 0 & 0 & 0 & 0 \\ t[5] & 0 & 0 & 0 & 0 & 0 & -\frac{-1+T_5}{T_5} & 0 & 0 \\ t[7] & 0 & 0 & 0 & 0 & 0 & 0 & -1 + T_7 & 0 \\ t[9] & 0 & -\frac{-1+T_9}{T_9} & 0 & 0 & 0 & 0 & 0 & 0 \\ t[11] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 + T_{11} \\ t[13] & -\frac{-1+T_{13}}{T_{13}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ t[15] & 0 & 0 & 0 & 0 & -1 + T_{15} & 0 & 0 & 0 \end{array} \right)$$

$\beta = \text{Times} @@ \text{GD}[K] /. \{\text{Ar}[\mathbf{x}_-, \mathbf{y}_-, +1] \Rightarrow \text{R}[\mathbf{x}, \mathbf{y}], \text{Ar}[\mathbf{x}_-, \mathbf{y}_-, -1] \Rightarrow \text{Rinv}[\mathbf{x}, \mathbf{y}]\};$

Table[

{k, ($\beta = \beta // \text{gm}[1, k, 1]$) // βForm , Collect[Last[β] /. t[i_] \Rightarrow 1, _h, Simplify]},

{k, 2, 2 Crossings[K]}

] // ColumnForm

$$\left\{ 2, \begin{array}{c} \left(\begin{array}{cccccccccc} 1 & h[1] & h[4] & h[6] & h[8] & h[10] & h[12] & h[14] & h[16] \\ t[1] & 0 & 0 & \frac{-1+T_1}{T_{13}} & 0 & 0 & 0 & 0 & 0 \\ t[3] & 0 & 0 & 0 & -\frac{-1+T_3}{T_3} & 0 & 0 & 0 & 0 \\ t[5] & 0 & 0 & 0 & 0 & 0 & -\frac{-1+T_5}{T_5} & 0 & 0 \\ t[7] & 0 & 0 & 0 & 0 & 0 & 0 & -1+T_7 & 0 \\ t[9] & 0 & -\frac{-1+T_9}{T_9} & 0 & 0 & 0 & 0 & 0 & 0 \\ t[11] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1+T_{11} \\ t[13] & -\frac{-1+T_{13}}{T_{13}} & 0 & \frac{(-1+T_1)(-1+T_{13})}{T_{13}} & 0 & 0 & 0 & 0 & 0 \\ t[15] & 0 & 0 & 0 & 0 & -1+T_{15} & 0 & 0 & 0 \end{array} \right) \\ , h[6] (-1 + T_1) \end{array} \right.$$

$$\left\{ 3, \begin{array}{c} \left(\begin{array}{cccccccccc} 1 & h[1] & h[4] & h[6] & h[8] & h[10] & h[12] & h[14] & h[16] \\ t[1] & 0 & 0 & \frac{-1+T_1}{T_{13}} & -\frac{-1+T_1}{T_1} & 0 & 0 & 0 & 0 \\ t[5] & 0 & 0 & 0 & 0 & 0 & -\frac{-1+T_5}{T_5} & 0 & 0 \\ t[7] & 0 & 0 & 0 & 0 & 0 & 0 & -1+T_7 & 0 \\ t[9] & 0 & -\frac{-1+T_9}{T_9} & 0 & 0 & 0 & 0 & 0 & 0 \\ t[11] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1+T_{11} \\ t[13] & -\frac{-1+T_{13}}{T_{13}} & 0 & \frac{(-1+T_1)(-1+T_{13})}{T_{13}} & 0 & 0 & 0 & 0 & 0 \\ t[15] & 0 & 0 & 0 & 0 & -1+T_{15} & 0 & 0 & 0 \end{array} \right) \\ , h[8] \left(-1 + \frac{1}{T_1}\right) \end{array} \right.$$

$$\left\{ 4, \begin{array}{c} \left(\begin{array}{cccccccccc} 1 & h[1] & h[6] & h[8] & h[10] & h[12] & h[14] & h[16] \\ t[1] & 0 & \frac{-1+T_1}{T_9 T_{13}} & -\frac{-1+T_1}{T_1 T_9} & 0 & 0 & 0 & 0 \\ t[5] & 0 & 0 & 0 & 0 & -\frac{-1+T_5}{T_5} & 0 & 0 \\ t[7] & 0 & 0 & 0 & 0 & 0 & -1+T_7 & 0 \\ t[9] & -\frac{-1+T_9}{T_9 T_{13}} & \frac{(-1+T_1)(-1+T_9)}{T_9 T_{13}} & -\frac{(-1+T_1)(-1+T_9)}{T_1 T_9} & 0 & 0 & 0 & 0 \\ t[11] & 0 & 0 & 0 & 0 & 0 & 0 & -1+T_{11} \\ t[13] & -\frac{-1+T_{13}}{T_{13}} & \frac{(-1+T_1)(-1+T_{13})}{T_{13}} & 0 & 0 & 0 & 0 & 0 \\ t[15] & 0 & 0 & 0 & -1+T_{15} & 0 & 0 & 0 \end{array} \right) \\ , h[8] \left(-1 + \frac{1}{T_1}\right) \end{array} \right.$$

$$\left\{ 5, \begin{array}{c} \left(\begin{array}{cccccccccc} 1 & h[1] & h[6] & h[8] & h[10] & h[12] & h[14] & h[16] \\ t[1] & 0 & \frac{-1+T_1}{T_9 T_{13}} & -\frac{-1+T_1}{T_1 T_9} & 0 & -\frac{-1+T_1}{T_1} & 0 & 0 \\ t[7] & 0 & 0 & 0 & 0 & 0 & -1+T_7 & 0 \\ t[9] & -\frac{-1+T_9}{T_9 T_{13}} & \frac{(-1+T_1)(-1+T_9)}{T_9 T_{13}} & -\frac{(-1+T_1)(-1+T_9)}{T_1 T_9} & 0 & 0 & 0 & 0 \\ t[11] & 0 & 0 & 0 & 0 & 0 & 0 & -1+T_{11} \\ t[13] & -\frac{-1+T_{13}}{T_{13}} & \frac{(-1+T_1)(-1+T_{13})}{T_{13}} & 0 & 0 & 0 & 0 & 0 \\ t[15] & 0 & 0 & 0 & -1+T_{15} & 0 & 0 & 0 \end{array} \right) \\ , h[8] \left(-1 + \frac{1}{T_1}\right) \end{array} \right.$$

$$\left\{ 6, \begin{array}{l} \frac{-1+T_1+T_9}{T_9 T_{13}} h[1] \quad h[8] \quad h[10] \quad h[12] \quad h[14] \quad h[16] \\ t[1] \quad \frac{(-1+T_1) T_1}{T_9 T_{13} (-1+T_1+T_9 T_{13})} \quad -\frac{(-1+T_1) T_{13}}{-1+T_1+T_9 T_{13}} \quad 0 \quad -\frac{(-1+T_1) T_9 T_{13}}{-1+T_1+T_9 T_{13}} \quad 0 \quad 0 \\ t[7] \quad 0 \quad 0 \quad 0 \quad 0 \quad -1+T_7 \quad 0 \\ t[9] \quad -\frac{-1+T_9}{-1+T_1+T_9 T_{13}} \quad -\frac{(-1+T_1) (-1+T_9) T_{13}}{T_1 (-1+T_1+T_9 T_{13})} \quad 0 \quad \frac{(-1+T_1)^2 (-1+T_9)}{T_1 (-1+T_1+T_9 T_{13})} \quad 0 \quad 0 \\ t[11] \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -1+T_{11} \\ t[13] \quad -\frac{T_9 (-1+T_{13})}{-1+T_1+T_9 T_{13}} \quad \frac{(-1+T_1)^2 (-1+T_{13})}{T_1 (-1+T_1+T_9 T_{13})} \quad 0 \quad \frac{(-1+T_1)^2 T_9 (-1+T_{13})}{T_1 (-1+T_1+T_9 T_{13})} \quad 0 \quad 0 \\ t[15] \quad 0 \quad 0 \quad -1+T_{15} \quad 0 \quad 0 \quad 0 \end{array} \right\}, h[\varepsilon]$$

$$\left\{ 7, \begin{array}{l} \frac{-1+T_1+T_9}{T_9 T_{13}} h[1] \quad h[8] \quad h[10] \quad h[12] \quad h[14] \quad h[16] \\ t[1] \quad \frac{(-1+T_1) T_1}{T_9 T_{13} (-1+T_1+T_9 T_{13})} \quad -\frac{(-1+T_1) T_{13}}{-1+T_1+T_9 T_{13}} \quad 0 \quad -\frac{(-1+T_1) T_9 T_{13}}{-1+T_1+T_9 T_{13}} \quad -1+T_1 \quad 0 \\ t[9] \quad -\frac{-1+T_9}{-1+T_1+T_9 T_{13}} \quad -\frac{(-1+T_1) (-1+T_9) T_{13}}{T_1 (-1+T_1+T_9 T_{13})} \quad 0 \quad \frac{(-1+T_1)^2 (-1+T_9)}{T_1 (-1+T_1+T_9 T_{13})} \quad 0 \quad 0 \\ t[11] \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -1+T_{11} \\ t[13] \quad -\frac{T_9 (-1+T_{13})}{-1+T_1+T_9 T_{13}} \quad \frac{(-1+T_1)^2 (-1+T_{13})}{T_1 (-1+T_1+T_9 T_{13})} \quad 0 \quad \frac{(-1+T_1)^2 T_9 (-1+T_{13})}{T_1 (-1+T_1+T_9 T_{13})} \quad 0 \quad 0 \\ t[15] \quad 0 \quad 0 \quad -1+T_{15} \quad 0 \quad 0 \quad 0 \end{array} \right\}, h[\varepsilon]$$

$$\left\{ 8, \begin{array}{l} \frac{-1+T_1+T_{13}-T_1 T_{13}+T_9 T_{13}}{T_9 T_{13}} h[1] \quad h[10] \quad h[12] \quad h[14] \\ t[1] \quad \frac{(-1+T_1) (-1+T_{13})}{T_9 T_{13} (1-T_1-T_{13}+T_1 T_{13}-T_9 T_{13})} \quad 0 \quad \frac{(-1+T_1) T_9 T_{13}}{T_1 (1-T_1-T_{13}+T_1 T_{13}-T_9 T_{13})} \quad -\frac{(-1+T_1) (-1+T_1+T_9 T_{13})}{T_1 (1-T_1-T_{13}+T_1 T_{13}-T_9 T_{13})} \\ t[9] \quad -\frac{-1+T_9}{-1+T_1+T_{13}-T_1 T_{13}+T_9 T_{13}} \quad 0 \quad \frac{(-1+T_1)^2 (-1+T_9)}{T_1 (1-T_1-T_{13}+T_1 T_{13}-T_9 T_{13})} \quad -\frac{(-1+T_1)^2 (-1+T_9) T_{13}}{T_1 (1-T_1-T_{13}+T_1 T_{13}-T_9 T_{13})} \\ t[11] \quad 0 \quad 0 \quad 0 \quad 0 \\ t[13] \quad -\frac{(-1+T_1-T_9) (-1+T_{13})}{1-T_1-T_{13}+T_1 T_{13}-T_9 T_{13}} \quad 0 \quad -\frac{(-1+T_1)^2 T_9 (-1+T_{13})}{T_1 (1-T_1-T_{13}+T_1 T_{13}-T_9 T_{13})} \quad \frac{(-1+T_1)^3 (-1+T_{13})}{T_1 (1-T_1-T_{13}+T_1 T_{13}-T_9 T_{13})} \\ t[15] \quad 0 \quad -1+T_{15} \quad 0 \quad 0 \end{array} \right\}$$

$$\left\{ 9, \begin{array}{l} \frac{-1+T_1+T_{13}}{T_1 T_{13}} h[1] \quad h[10] \quad h[12] \quad h[14] \\ t[1] \quad -\frac{(-1+T_1) (-1+T_{13}+T_1 T_{13})}{T_1 T_{13} (-1+T_1+T_{13})} \quad 0 \quad -\frac{(-1+T_1) (-1+2 T_1-T_1^2+T_{13}-T_1 T_{13}+T_1^2 T_{13})}{T_1 (-1+T_1+T_{13})} \quad \frac{(-1+T_1) (-1+T_1+T_{13}-T_1 T_{13}+T_1^2 T_{13})}{T_1 (-1+T_1+T_{13})} \\ t[11] \quad 0 \quad 0 \quad 0 \quad 0 \\ t[13] \quad -\frac{-1+T_{13}}{-1+T_1+T_{13}} \quad 0 \quad \frac{(-1+T_1)^2 (-1+T_{13})}{-1+T_1+T_{13}} \quad -\frac{(-1+T_1)^3 (-1+T_{13})}{T_1 (-1+T_1+T_{13})} \\ t[15] \quad 0 \quad -1+T_{15} \quad 0 \quad 0 \end{array} \right\}$$

$$\left\{ 10, \begin{array}{l} \frac{-1+T_1+T_{13}}{T_1 T_{13}} h[1] \quad h[12] \quad h[14] \\ t[1] \quad -\frac{(-1+T_1) (-1+T_{13}+T_1 T_{13}) T_{15}}{T_1 T_{13} (-1+T_1+T_{13})} \quad -\frac{(-1+T_1) (-1+2 T_1-T_1^2+T_{13}-T_1 T_{13}+T_1^2 T_{13}) T_{15}}{T_1 (-1+T_1+T_{13})} \quad \frac{(-1+T_1) (-1+T_1+T_{13}-T_1 T_{13}+T_1^2 T_{13})}{T_1 (-1+T_1+T_{13})} \\ t[11] \quad 0 \quad 0 \quad 0 \\ t[13] \quad -\frac{-1+T_{13}}{-1+T_1+T_{13}} \quad \frac{(-1+T_1)^2 (-1+T_{13})}{-1+T_1+T_{13}} \quad -\frac{(-1+T_1)^3 (-1+T_{13})}{T_1 (-1+T_1+T_{13})} \\ t[15] \quad \frac{T_1 (-1+T_{15})}{-1+T_1+T_{13}} \quad \frac{(-1+T_1) (-1+2 T_1-T_1^2+T_{13}-T_1 T_{13}+T_1^2 T_{13}) (-1+T_{15})}{T_1 (-1+T_1+T_{13})} \quad -\frac{(-1+T_1) (-1+T_1+T_{13}-T_1 T_{13}+T_1^2 T_{13})}{T_1 (-1+T_1+T_{13})} \end{array} \right\}$$

$$\left\{ 11, \begin{array}{l} \frac{-1+T_1+T_{13}}{T_1 T_{13}} h[1] \quad h[12] \quad h[14] \\ t[1] \quad -\frac{(-1+T_1) (-1+T_{13}+T_1 T_{13}) T_{15}}{T_1 T_{13} (-1+T_1+T_{13})} \quad -\frac{(-1+T_1) (-1+2 T_1-T_1^2+T_{13}-T_1 T_{13}+T_1^2 T_{13}) T_{15}}{T_1 (-1+T_1+T_{13})} \quad \frac{(-1+T_1) (-1+T_1+T_{13}-T_1 T_{13}+T_1^2 T_{13})}{T_1 (-1+T_1+T_{13})} \\ t[13] \quad -\frac{-1+T_{13}}{-1+T_1+T_{13}} \quad \frac{(-1+T_1)^2 (-1+T_{13})}{-1+T_1+T_{13}} \quad -\frac{(-1+T_1)^3 (-1+T_{13})}{T_1 (-1+T_1+T_{13})} \\ t[15] \quad \frac{T_1 (-1+T_{15})}{-1+T_1+T_{13}} \quad \frac{(-1+T_1) (-1+2 T_1-T_1^2+T_{13}-T_1 T_{13}+T_1^2 T_{13}) (-1+T_{15})}{T_1 (-1+T_1+T_{13})} \quad -\frac{(-1+T_1) (-1+T_1+T_{13}-T_1 T_{13}+T_1^2 T_{13})}{T_1 (-1+T_1+T_{13})} \end{array} \right\}$$

$$\left\{ \begin{array}{l}
 12, \left(\begin{array}{l}
 -\frac{T_1 - T_1^2 - T_1 T_{13} + T_{15} - 3 T_1 T_{15} + 3 T_1^2 T_{15} - T_1^3 T_{15} - T_{13} T_{15} + 2 T_1 T_{13} T_{15} - 2 T_1^2 T_{13} T_{15} + T_1^3 T_{13} T_{15}}{T_1^2 T_{13}} \quad h \\
 t[1] \quad \frac{(-1+T_1) T_{15} (-T_1+T_1 T_{13}+T_1^2 T_{13}-T_{15}+2 T_1 T_{13} (T_1-T_1^2-T_1 T_{13}+T_{15}-3 T_1 T_{15}+3 T_1^2 T_{15}-T_1^3 T_{15}-T_1^3 T_{13} T_{15}+T_1^3 T_{13} T_{15}+T_1^3 T_{15}-T_1^3 T_{13} T_{15})}{T_1^2 T_{13} (T_1-T_1^2-T_1 T_{13}+T_{15}-3 T_1 T_{15}+3 T_1^2 T_{15}-T_1^3 T_{15}-T_1^3 T_{13} T_{15}+T_1^3 T_{13} T_{15}+T_1^3 T_{15}-T_1^3 T_{13} T_{15})} \\
 t[13] \quad -\frac{(-1+T_{13}) (-T_1-T_{15}+T_1 T_{13}+T_1^2 T_{13}-T_{15}+2 T_1 T_{13} (T_1-T_1^2-T_1 T_{13}+T_{15}-3 T_1 T_{15}+3 T_1^2 T_{15}-T_1^3 T_{15}-T_1^3 T_{13} T_{15}+T_1^3 T_{13} T_{15}+T_1^3 T_{15}-T_1^3 T_{13} T_{15})}{T_1-T_1^2-T_1 T_{13}+T_{15}-3 T_1 T_{15}+3 T_1^2 T_{15}-T_1^3 T_{15}-T_1^3 T_{13} T_{15}+T_1^3 T_{13} T_{15}+T_1^3 T_{15}-T_1^3 T_{13} T_{15})} \\
 t[15] \quad -\frac{T_1^2 (-1+T_{13}) (-T_1-T_{15}+T_1 T_{13}+T_1^2 T_{13}-T_{15}+2 T_1 T_{13} (T_1-T_1^2-T_1 T_{13}+T_{15}-3 T_1 T_{15}+3 T_1^2 T_{15}-T_1^3 T_{15}-T_1^3 T_{13} T_{15}+T_1^3 T_{13} T_{15}+T_1^3 T_{15}-T_1^3 T_{13} T_{15})}{T_1-T_1^2-T_1 T_{13}+T_{15}-3 T_1 T_{15}+3 T_1^2 T_{15}-T_1^3 T_{15}-T_1^3 T_{13} T_{15}+T_1^3 T_{13} T_{15}+T_1^3 T_{15}-T_1^3 T_{13} T_{15})}
 \end{array} \right. \\
 13, \left(\begin{array}{l}
 -\frac{T_1-2 T_1^2+T_{15}-4 T_1 T_{15}+5 T_1^2 T_{15}-3 T_1^3 T_{15}+T_1^4 T_{15}}{T_1^3} \quad h[1] \\
 t[1] \quad -\frac{(-1+T_1) (-T_1^4+T_1 T_{15}-T_1^2 T_{15}-2 T_1^3 T_{15}+2 T_1^4 T_{15}-2 T_1^5 T_{15}+T_1^6 T_{15}+T_{15}^2-3 T_1 T_{15}^2+2 T_1^2 T_{15}^2-T_1^3 T_{15}^2+T_1^4 T_{15}^2-T_1^5 T_{15}^2+T_1^6 T_{15}^2-T_1^7 T_{15}^2+T_1^8 T_{15}^2-T_1^9 T_{15}^2+T_1^{10} T_{15}^2-T_1^{11} T_{15}^2+T_1^{12} T_{15}^2-T_1^{13} T_{15}^2+T_1^{14} T_{15}^2-T_1^{15} T_{15}^2)}{T_1^3 (T_1-2 T_1^2+T_{15}-4 T_1 T_{15}+5 T_1^2 T_{15}-3 T_1^3 T_{15}+T_1^4 T_{15}+T_1^5 T_{15}-2 T_1^6 T_{15}+T_1^7 T_{15}-2 T_1^8 T_{15}+T_1^9 T_{15}-2 T_1^{10} T_{15}+T_1^{11} T_{15}-2 T_1^{12} T_{15}+T_1^{13} T_{15}-2 T_1^{14} T_{15}+T_1^{15} T_{15})} \\
 t[15] \quad -\frac{T_1^2 (-1+T_{15}) (-T_1-T_{15}+T_1 T_{13}+T_1^2 T_{13}-T_{15}+2 T_1 T_{13} (T_1-T_1^2-T_1 T_{13}+T_{15}-3 T_1 T_{15}+3 T_1^2 T_{15}-T_1^3 T_{15}-T_1^3 T_{13} T_{15}+T_1^3 T_{13} T_{15}+T_1^3 T_{15}-T_1^3 T_{13} T_{15})}{T_1-2 T_1^2+T_{15}-4 T_1 T_{15}+5 T_1^2 T_{15}-3 T_1^3 T_{15}+T_1^4 T_{15}+T_1^5 T_{15}-2 T_1^6 T_{15}+T_1^7 T_{15}-2 T_1^8 T_{15}+T_1^9 T_{15}-2 T_1^{10} T_{15}+T_1^{11} T_{15}-2 T_1^{12} T_{15}+T_1^{13} T_{15}-2 T_1^{14} T_{15}+T_1^{15} T_{15})}
 \end{array} \right. \\
 14, \left(\begin{array}{l}
 -\frac{1-3 T_1+4 T_1^2-4 T_1^3+T_1^4-T_{15}+4 T_1 T_{15}-7 T_1^2 T_{15}+7 T_1^3 T_{15}-4 T_1^4 T_{15}+T_1^5 T_{15}}{T_1^3} \quad h[1] \\
 t[1] \quad -\frac{(-1+T_1) (-T_1^4+T_{15}-2 T_1 T_{15}+2 T_1^2 T_{15}-3 T_1^3 T_{15}+2 T_1^4 T_{15}-2 T_1^5 T_{15}+T_1^6 T_{15}+T_{15}^2-3 T_1 T_{15}^2+2 T_1^2 T_{15}^2-T_1^3 T_{15}^2+T_1^4 T_{15}^2-T_1^5 T_{15}^2+T_1^6 T_{15}^2-T_1^7 T_{15}^2+T_1^8 T_{15}^2-T_1^9 T_{15}^2+T_1^{10} T_{15}^2-T_1^{11} T_{15}^2+T_1^{12} T_{15}^2-T_1^{13} T_{15}^2+T_1^{14} T_{15}^2-T_1^{15} T_{15}^2)}{T_1^3 (1-3 T_1+4 T_1^2-4 T_1^3+T_1^4-T_{15}+4 T_1 T_{15}-7 T_1^2 T_{15}+7 T_1^3 T_{15}-4 T_1^4 T_{15}+T_1^5 T_{15}-2 T_1^6 T_{15}+T_1^7 T_{15}-2 T_1^8 T_{15}+T_1^9 T_{15}-2 T_1^{10} T_{15}+T_1^{11} T_{15}-2 T_1^{12} T_{15}+T_1^{13} T_{15}-2 T_1^{14} T_{15}+T_1^{15} T_{15})} \\
 t[15] \quad \frac{(1-3 T_1+3 T_1^2-3 T_1^3+T_1^4) (-1+T_{15}) (-T_1-T_{15}+T_1 T_{13}+T_1^2 T_{13}-T_{15}+2 T_1 T_{13} (T_1-T_1^2-T_1 T_{13}+T_{15}-3 T_1 T_{15}+3 T_1^2 T_{15}-T_1^3 T_{15}-T_1^3 T_{13} T_{15}+T_1^3 T_{13} T_{15}+T_1^3 T_{15}-T_1^3 T_{13} T_{15})}{1-3 T_1+4 T_1^2-4 T_1^3+T_1^4-T_{15}+4 T_1 T_{15}-7 T_1^2 T_{15}+7 T_1^3 T_{15}-4 T_1^4 T_{15}+T_1^5 T_{15}-2 T_1^6 T_{15}+T_1^7 T_{15}-2 T_1^8 T_{15}+T_1^9 T_{15}-2 T_1^{10} T_{15}+T_1^{11} T_{15}-2 T_1^{12} T_{15}+T_1^{13} T_{15}-2 T_1^{14} T_{15}+T_1^{15} T_{15})}
 \end{array} \right. \\
 15, \left(\begin{array}{l}
 -\frac{1-4 T_1+8 T_1^2-11 T_1^3+8 T_1^4-4 T_1^5+T_1^6}{T_1^3} \quad h[1] \quad h[16] \\
 t[1] \quad -\frac{-1+T_1}{T_1} \quad -1+T_1 \end{array} \right), \quad h[1] \left(-1 + \frac{1}{T_1} \right) + h[16] (-1 + T_1) \} \\
 16, \left(\begin{array}{l}
 -\frac{1-4 T_1+8 T_1^2-11 T_1^3+8 T_1^4-4 T_1^5+T_1^6}{T_1^3} \\
 t[1] \end{array} \right), \quad 0 \}
 \end{array} \right.$$

β

$$B \left[-\frac{1-4 T_1+8 T_1^2-11 T_1^3+8 T_1^4-4 T_1^5+T_1^6}{T_1^3}, 0 \right]$$

{Alexander[K = Knot[8, 17]][X], Draw[GD[K]]}

$$\left\{ 11 - \frac{1}{X^3} + \frac{4}{X^2} - \frac{8}{X} - 8X + 4X^2 - X^3, \right.$$

```

β = Times @@ GD[K] /. {Ar[x_, y_, +1] => R[x, y], Ar[x_, y_, -1] => Rinv[x, y]};
indices = Range[2 Crossings[K]];

```

```
Table[
```

```
  j = RandomInteger[{1, k - 1}];
```

```
  β = β // gm[indices[[j]], indices[[j + 1]], indices[[j]]];
```

```
  indices = Delete[indices, j + 1];
```

```
  {indices, β // βForm; Collect[Last[β] /. t[i_] => 1, _h, Simplify[1 + #] &]},
```

```
  {k, 2 Crossings[K], 2, -1}
```

```
] // ColumnForm
```

```

{ {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16}, h[6] T1 +  $\frac{h[8]}{T_3} + \frac{h[12]}{T_5} + h[13] T_7 + \frac{h[4]}{T_9} + h[16]$  }
{ {1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 15, 16}, h[6] T1 +  $\frac{h[7]}{T_3} + \frac{h[12]}{T_5} + h[13] T_7 + \frac{h[4]}{T_9} + h[16] T_{11}$  }
{ {1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 15, 16},  $\frac{h[6] T_1}{T_3} + \frac{h[12]}{T_5} + h[13] T_6 + \frac{h[4]}{T_9} + h[16] T_{11} + \frac{h[2]}{T_{13}} + h[10]$  }
{ {1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 15, 16},  $\frac{h[6] T_1}{T_3} + \frac{h[12]}{T_5} + \frac{h[4]}{T_6} + h[13] T_6 + h[16] T_{11} + \frac{h[2]}{T_{13}} + h[10]$  }
{ {1, 3, 4, 5, 6, 10, 11, 12, 13, 15, 16},  $\frac{h[6] T_1}{T_3} + \frac{h[12]}{T_5} + \frac{h[4]}{T_6} + h[13] T_6 + h[16] T_{11} + \frac{h[1]}{T_{13}} + h[10] T_1$  }
{ {1, 3, 4, 5, 6, 11, 12, 13, 15, 16},  $\frac{h[12]}{T_5} + \frac{h[4]}{T_6} + h[13] T_6 + h[16] T_{11} + \frac{h[1]}{T_{13}} + \frac{h[6] T_1 T_{15}}{T_3}$  }
{ {1, 3, 4, 6, 11, 12, 13, 15, 16},  $\frac{h[12]}{T_4} + \frac{h[4]}{T_6} + h[13] T_6 + h[16] T_{11} + \frac{h[1]}{T_{13}} + \frac{h[6] T_1 T_{15}}{T_3}$  }
{ {1, 4, 6, 11, 12, 13, 15, 16},  $\frac{h[12]}{T_4} + \frac{h[4]}{T_6} + h[13] T_6 + h[16] T_{11} + \frac{h[1]}{T_{13}} + h[6] T_{15}$  }
{ {1, 4, 11, 12, 13, 15, 16},  $\frac{h[12]}{T_4} + h[13] T_4 + h[16] T_{11} + \frac{h[1]}{T_{13}} + \frac{h[4] T_{15}}{T_4}$  }
{ {1, 4, 11, 13, 15, 16},  $\frac{h[11]}{T_4} + h[13] T_4 + h[16] T_{11} + \frac{h[1]}{T_{13}} + \frac{h[4] T_{15}}{T_4}$  }
{ {1, 4, 13, 15, 16}, h[13] T4 + h[16] T4 +  $\frac{h[1]}{T_{13}} + \frac{h[4] T_{15}}{T_4^2}$  }
{ {1, 4, 15, 16},  $\frac{h[1]}{T_4} + h[16] T_4 + \frac{h[4] T_{15}}{T_4}$  }
{ {1, 4, 16}, h[4] +  $\frac{h[1]}{T_4} + h[16] T_4$  }
{ {1, 4},  $\frac{h[1]}{T_4} + h[4] T_4$  }
{ {1}, 1 }

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β // βForm
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$$\begin{pmatrix} -\frac{1-4 T_1+8 T_1^2-11 T_1^3+8 T_1^4-4 T_1^5+T_1^6}{T_1^2} & h[1] \\ t[1] & 0 \end{pmatrix}$$